Title: On some ideal related to the ideal (v0)

Author: Piotr Kalemba

Citation style: Kalemba Piotr. (2015). On some ideal related to the ideal (v0). “Open Mathematics” (Vol. 13, Iss. 1 (2015), s. 421-425), doi 10.1515/math-2015-0039.

DE GRUYTER OPEN Open Math. 2015; 13: 421–425

Open Mathematics **Open Access**

Research Article

Piotr Kalemba*

On some ideal related to the ideal (v^0)

DOI 10.1515/math-2015-0039
Received September 12, 2014; accepted April 3, 2015.

Abstract: The ideal (v^0) is known in the literature and is naturally linked to the structure $[\omega]^\omega$. We consider some natural counterpart of the ideal (v^0) related in an analogous way to the structure Dense($\mathbb{Q}$) and investigate its combinatorial properties. By the use of the notion of ideal type we prove that under CH this ideal is isomorphic to (v^0).

Keywords: Ideal (v^0), Ideal isomorphism, Ideal type, Continuum hypothesis

MSC: 03E35, 03E15

1 Introduction

A family of sets $\mathcal{I}$ is called an ideal if it is closed under both subsets and finite unions of its elements. Additionally, we assume throughout the paper, that all singletons are members of an ideal and the union of an ideal is of the cardinality continuum. We say that two ideals $\mathcal{I}$ and $\mathcal{J}$ are isomorphic if there exists a bijection $f: \bigcup \mathcal{I} \to \bigcup \mathcal{J}$ such that for any set X, it holds X is a member of $\mathcal{I}$ if and only if its image $f(X)$ is a member of $\mathcal{J}$. Let $\mathcal{R} \subseteq \mathcal{P}(\bigcup \mathcal{I})$ we say that the poset (partially ordered set) $(\mathcal{R}, \subseteq)$ is a MB-representation (after Marczewski, Burstin) for the ideal $\mathcal{I}$ whenever for any X it holds $X \in \mathcal{I}$ if and only if for any $U \in \mathcal{R}$ there exists $V \in \mathcal{R}$ such that $V \subseteq U$ and V is disjoint with X.

The ideal (v^0) is known in the literature and corresponds to the Silver forcing, compare [3]. In [4] it was defined by a use of special families that we call segments. A segment is a family $< D, E >= \{X : D \subseteq X \subseteq E\}$, where $D \subset E$ and $E \setminus D$ is infinite. The ideal (v^0) is a family of all $\mathcal{X} \subseteq \mathcal{P}(\omega)$ such that for any segment $< A, B >$ $(B \subseteq \omega)$ there is a segment $< C, D > \subseteq < A, B >$ such that $< C, D > \cap \mathcal{X} = \emptyset$. So the family of segments $\{< A, B >: B \subseteq \omega\}$ is a MB-representation for this ideal, and any segment of this MB-representation satisfies $B \setminus A \in [\omega]^\omega$ thus the ideal (v^0) is naturally related to the structure $[\omega]^\omega$. Another natural structure of the cardinality continuum, consisting of infinite countable sets and ordered by inclusion is the family Dense($\mathbb{Q}$) $= \{D \subseteq \mathbb{Q} : D \text{ is dense}\}$. In [1] there were investigated similarities and differences between combinatorial properties of structures $[\omega]^\omega$ and Dense($\mathbb{Q}$) and some ideals associated to them. So, one may ask for a natural counterpart of (v^0) in relation to the structure Dense($\mathbb{Q}$). We would like to propose such an ideal, which we denote (d^0).

In order to do this let us first consider the family $\mathbb{D}$ of all segments $\{< D, E >: E \subseteq \mathbb{Q}$ and $E \setminus D$ is a dense subset of $\mathbb{Q}\}$. By (d^0) we understand the family of all $\mathcal{Q} \subseteq \mathcal{P}(\mathbb{Q})$ such that any segment from $\mathbb{D}$ contains a segment from $\mathbb{D}$ which is disjoint with $\mathcal{Q}$. In other words, the poset $(\mathbb{D}, \subseteq)$ is a MB-representation for the ideal (d^0). Note that if $< A, B >$ is a member of the MB-representation for the ideal (v^0), then $B \setminus A$ belongs to $[\omega]^\omega$, whereas if $< D, E > \in \mathbb{D}$ then $E \setminus D$ belongs to Dense($\mathbb{Q}$). The main goal of this paper is to show that the ideals (v^0) and (d^0) are isomorphic under CH (by CH we understand the continuum hypothesis, i.e. $2^\omega = \omega_1$).

***Corresponding Author: Piotr Kalemba:** Institute of Mathematics, University of Silesia in Katowice, Poland,
E-mail: piotr.kalemba@us.edu.pl

2 The ideal (d^0) is a σ-ideal

In this section it will be convenient to use some auxiliary notation, if Q is a finite subset of the rationals, then $< A, B >_Q = < A, B \setminus (Q \setminus A) >$, i.e. $< A, B >_Q$ is such a segment that $X \in < A, B >_Q$ if and only if $X \in < A, B >$ and $X \cap Q = A \cap Q$.

Lemma 2.1. *The ideal* (d^0) *is a* σ*-ideal.*

Proof. Let $\mathcal{X}_0, \mathcal{X}_1, \ldots$ be increasing elements of (d^0), $< A, B > \in \mathbb{D}$ and $\{U_n : n < \omega\}$ be a basis of the rational numbers. We are going to define inductively for each natural number n, rational numbers $q_0, q_1, \ldots, q_n$ and sets $A \subseteq A_0 \subseteq A_1 \subseteq \ldots \subseteq A_n \subseteq B_n \subseteq \ldots \subseteq B_1 \subseteq B_0 \subseteq B$ such that $q_n \in U_n \cap (B_n \setminus A_n)$, $Q_n = \{q_0, q_1, \ldots q_n\} \subseteq B_n \setminus A_n$, $B_n \setminus A_n$ is a dense set and $< A_n \cup x, B_n >_{Q_n}$ is disjoint with $\mathcal{X}_n$ for any $x \subseteq Q_n$.

Take $q_0 \in U_0 \cap (B \setminus A)$, put $Q_0 = \{q_0\}$ and choose $< A_0^0, B_0^0 > \subseteq < A, B >_{Q_0} \setminus \mathcal{X}_0$. Next choose a segment $< C, D > \in \mathbb{D}$, $< C, D > \subseteq < A_0^0 \cup \{q_0\}, B_0^0 >_{Q_0} \setminus \mathcal{X}_0$ and put $A_0^1 = C \setminus \{q_0\}$ and $B_0^1 = D$. Then $A_0^1 \supseteq A_0^0$ and $B_0^1 \subseteq B_0^0 \cup Q_0$ and $B_0^1 \setminus A_0^1$ is dense and contains Q_0 and, moreover, $< A_0^1 \cup \{q_0\}, B_0^1 >_{Q_0}$ is disjoint with $\mathcal{X}_0$. Let A_0 and B_0 be A_0^1 and B_0^1 respectively.

The inductive step is similar to the 0'th. Assume that sets A_n, B_n and points from Q_{n-1} have been already defined. Take any $q_n \in U_n \cap (B_n \setminus A_n)$ and enumerate all subsets of $Q_n = Q_{n-1} \cup \{q_n\}$ into $x_1, x_2, \ldots, x_m$. Choose $A_n^1 \supseteq A_n$ and $B_n^1 \subseteq B_n \cup Q_n$ such that $B_n^1 \setminus A_n^1$ is dense, contains Q_n and, moreover, $< A_n^1 \cup x_1, B_n^1 >_{Q_n} \cap \mathcal{X}_n = \emptyset$. Now, assume that A_n^{k-1} and B_n^{k-1} have been determined and choose $A_n^k \supseteq A_n^{k-1}$ and $B_n^k \subseteq B_n^{k-1} \cup Q_n$ such that set $B_n^k \setminus A_n^k$ is dense and contains Q_n and $< A_n^k \cup x_k, B_n^k >_{Q_n}$ is disjoint with $\mathcal{X}_n$. Finally, putting $A_{n+1} = A_n^m$ and $B_{n+1} = B_n^m$ we are done with the induction.

Let $A_\infty = \bigcup_n A_n$ and $B_\infty = A_\infty \cup \{q_n : n < \omega\}$. Then the segment $< A_\infty, B_\infty > \subseteq < A, B >$ and is disjoint with each $\mathcal{X}_n$. Indeed, suppose $X \in < A_\infty, B_\infty > \cap \mathcal{X}_n$. Let $x = X \cap Q_n$, then $X \in < A_n \cup x, B_n >_{Q_n}$ which contradicts an inductive condition. □

3 The poset $(\mathbb{D}^*, \subseteq)$

To investigate the structure of the ideal (d^0) we will need the notion of Base Tree, which first appeared in paper [2]. We will present this notion in a simplified version which is sufficient to our purpose.

A Base Tree in a poset $(\mathcal{R}, \subseteq)$ is a family $\Theta = \{u_\alpha : \alpha < \mathfrak{h}(\mathcal{R})\}$ such that: $\bigcup \Theta$ is a tree, ordered by $\supseteq$, of height $\mathfrak{h}(\mathcal{R})$; each u_α is a level of this tree and is a maximal antichain in $\mathcal{R}$; each $U \in \bigcup \Theta$ has continuum many immediate successors; for every $V \in \mathcal{R}$ there is a $U \in \bigcup \Theta$ such that $U \subseteq V$. Here $\mathfrak{h}(\mathcal{R})$ stands for the minimal cardinality of a family of maximal antichains in $\mathcal{R}$ with no common refinement.

The following theorem is due to B. Balcar, J. Pelant and P. Simon, compare[2, 7]: *An atomless poset* $(\mathcal{R}, \subseteq)$ *of cardinality continuum has a Base Tree, whenever it has so called Tree Property (TP) i.e. it is separative,* σ*-closed and homogeneous in hight.*

Let me recall that a poset $(\mathcal{R}, \subseteq)$ is *separative* if for any $U, V \in \mathcal{R}$ such that $U \nsubseteq V$ there is some $W \subseteq U$ incompatible with V (i.e. there is no element of $\mathcal{R}$ contained in $W \cap V$). It is σ*-closed* if any countable decreasing sequence of elements of $\mathcal{R}$ has a lower bound in $\mathcal{R}$. A poset $(\mathcal{R}, \subseteq)$ is *homogeneous in height* if $\mathfrak{h}(\mathcal{R}) = \mathfrak{h}(\mathcal{R} \downarrow U)$ for every $U \in \mathcal{R}$, where $\mathcal{R} \downarrow U = \{V \in \mathcal{R} : V \subseteq U\}$.

The family $\mathbb{D}$ does not have Tree Property, so in order to apply the apparatus of Base Tree to an investigation of the ideal (d^0) we need a slightly amended MB-representation for this ideal. Instead of the family $\mathbb{D}$ we are going to use the family $\mathbb{D}^*$ consisting of all sets $< C, D >^* = \{X : C \subseteq^* X \subseteq^* D\}$ for $< C, D > \in \mathbb{D}$ (here the relation $\subseteq^*$ stands for almost inclusion, i.e. $C \subseteq^* D$ if and only if $C \setminus D$ is finite). It turns out that the family $\mathbb{D}^*$ remains a MB-representation for (d^0) and, moreover, it has Tree Property.

Proposition 3.1. *The poset* $(\mathbb{D}^*, \subseteq)$ *is a MB-representation for the ideal* (d^0).

Proof. Assume $\mathcal{Q} \in (d^0)$ and $< D, E >^* \in \mathbb{D}^*$. Then, by Lemma 2.1, $\mathcal{Q}^* = \{X \subseteq \mathbb{Q} : X =^* Y \text{ for some } Y \in \mathcal{Q}\}$ is a member of (d^0), too. Choose $< D_1, E_1 > \subseteq < D, E >$ such that $< D_1, E_1 > \in \mathbb{D}$ and $< D_1, E_1 > \cap \mathcal{Q}^* = \emptyset$. It follows that $< D_1, E_1 >^* \cap \mathcal{Q}^* = \emptyset$, hence $\mathcal{Q}$ is a member of the ideal whose MB-representation is $\mathbb{D}^*$. The converse implication is straightforward. □

Proposition 3.2. *The poset* $(\mathbb{D}^*, \subseteq)$ *has Tree Property.*

Proof. To see that the poset is atomless it is enough to note that any dense subset of the rational numbers can be split into two disjoint dense subsets. We will show subsequently that $\mathbb{D}^*$ is σ-closed, separative and homogeneous in height:

(1) Let $\{< D_n, E_n >^* : n \in \omega\}$ be a decreasing sequence of elements of $\mathbb{D}^*$. Take a set X such that $D_1 \subseteq^* D_2 \subseteq^* \ldots \subseteq^* X \subseteq^* \ldots \subseteq^* E_2 \subseteq^* E_1$. Next, take a dense set G such that $G \subseteq^* E_n \setminus D_n$ for any $n < \omega$. Then $< X \setminus G, X \cup G >^* \in \mathbb{D}^*$ and $< X \setminus G, X \cup G >^* \subseteq < D_n, E_n >^*$ for any $n \in \omega$.

(2) Let $< A, V >^* \not\subseteq < B, W >^*$ be elements of $\mathbb{D}^*$. We may assume that families $< A, V >^*$ and $< B, W >^*$ are compatible, so $< A, V >^* \cap < B, W >^* = < A \cup B, V \cap W >^* \in \mathbb{D}^*$. Assume $V = A \cup D$ and $W = B \cup G$, where D and G are dense sets disjoint respectively with A and B. We have $V \cap W \setminus (A \cup B) = D \cap G$, therefore $D \cap G$ is a dense set.

Moreover, $B \not\subseteq^* A$ or $V \not\subseteq^* W$. It is easy to check that in the first case, a desired member of $\mathbb{D}^*$ is $< A, A \cup (D \cap G) >^* \subseteq < A, A \cup D >^*$ and in the other case, member $< A \cup (V \setminus W), V >^* \subseteq < A, V >^*$ is as desired.

(3) Take any segment $< A, B > \in \mathbb{D}$. Set B is dense, so by Sierpinski characterization of the rational numbers, there exists $h : B \longrightarrow \mathbb{Q}$ which is a homeomorphism. Then the function $H : (\mathbb{D} \downarrow < A, B >, \subseteq) \longrightarrow (\mathbb{D}, \subseteq)$ defined as $H(< X, Y >) = < h[X], h[Y] >$ is an isomorphism of these orders. The function $H^* : (\mathbb{D}^* \downarrow < A, B >^*, \subseteq) \longrightarrow (\mathbb{D}^*, \subseteq)$, defined naturally as $H^*(< X, Y >^*) = (H(< X, Y >))^*$, witnesses homogeneity. □

Observe that we have a counterpart for a Haussdorf gap, i.e. there is a pair of sequences $(\{D_\alpha : \alpha < \omega_1\}, \{E_\alpha : \alpha < \omega_1\})$ of dense subsets of the rationals such that for any $\alpha < \beta < \omega_1$, $D_\alpha \subseteq^* D_\beta$, $E_\beta \subseteq^* E_\alpha$, $E_\beta \setminus D_\alpha$ is dense and there is no set X with $D_\alpha \subseteq^* X \subseteq^* E_\alpha$ for any α. Indeed, take any Hausdorf gap $(\{A_\alpha : \alpha < \omega_1\}, \{B_\alpha : \alpha < \omega_1\})$ on the natural numbers. Let D and G be two disjoint dense sets disjoint with the natural numbers. Now, the families $\{A_\alpha \cup D : \alpha < \omega_1\}$ and $\{B_\alpha \cup D \cup G : \alpha < \omega_1\}$ form a desired gap. This means that in $(\mathbb{D}^*, \subseteq)$ there exist maximal (well-ordered) chains of the length ω_1.

4 Ideal type

The ideal (v^0) was defined originally as a family of subsets of $[\omega]^\omega$, however, to compare it with the ideal (d^0) it is convenient to see (v^0) as a family of subsets of the rational numbers. We mean the ideal defined by the same definition as (v^0) but on $\mathcal{P}(\mathbb{Q})$ instead of on $\mathcal{P}(\omega)$. Up to isomorphism it is of course the same ideal as (v^0), but it turns out to be different (as a family) from the ideal (d^0), because we have the following fact:

Fact 4.1. *Segment* $< E, F > \in (d^0)$ *if and only if* $F \setminus E$ *is not dense.*

Proof. Of course we need to prove if direction only. Let U be a nonempty open set such that $U \cap (F \setminus E) = \emptyset$. Take any segment $< A, B > \in \mathbb{D}$. If $[U \cap (B \setminus A)] \setminus E \neq \emptyset$ and p is in this difference, then the segment $< A \cup \{p\}, B >$ is disjoint with $< E, F >$. If $U \cap (B \setminus A) \subseteq E$ pick out some $p \in U \cap (B \setminus A)$, then the segment $< A, B \setminus \{p\} >$ is disjoint with $< E, F >$, too. □

The next lemma refers to the notion of ideal type. This concept has been introduced by Plewik in [6] and is based on the idea of Base Tree. It is a triple of cardinal numbers which is associated to an ideal. It can be determined if the ideal is related in some way to a suitable Base Tree in the family which is a MB-representation for this ideal (the tree cardinals are parameters of this Base Tree). What is important, it characterizes the ideal up to isomorphism, namely we have the following theorem, Theorem 1 in [6]:

Two ideals with the same ideal type are isomorphic.

For the reader's convenience we will bring up here the definition of ideal type and relate its notation to the notation used in the proof of the next Lemma.

Let $\mathcal{I}$ be an ideal and $\mathcal{M}$ a family of subsets of the union $\bigcup \mathcal{I}$ such that for each finite family of members of $\mathcal{M}$ their intersection does not belong to $\mathcal{I}$. We consider the following two conditions:

(1) For each set X from $\mathcal{I}$ the difference $\bigcup \mathcal{I} \setminus \bigcup\{U \in \bigcup \mathcal{M} : U \cap X = \emptyset\}$ belongs to $\mathcal{I}$.
(2) Any nowhere dense set in the topology generated by $\mathcal{M}$ belongs to $\mathcal{I}$.

Now, let λ, τ be cardinal numbers. A family Γ of pairwise disjoint families of subsets which are contained in the union $\bigcup \mathcal{I}$ is called a λ-base tree of size τ associated to $\mathcal{I}$ if τ is a regular cardinal number, $\Gamma = \{v_\alpha : \alpha < \tau\}$, where $v_0 = \{\bigcup \mathcal{I}\}$, the family $\mathcal{M} = \bigcup \Gamma$ and $\mathcal{I}$ fulfills conditions (1) and (2), $\mathcal{M}$ does not meet $\mathcal{I}$ and if $\alpha < \beta$, then the family v_β refines the family v_α and each set from v_α contains exactly λ sets from v_β.

A chain is a family linearly ordered by inclusion. An ideal $\mathcal{I}$ has ideal type (λ, τ, γ) if it has a λ-base tree $\Gamma = \{v_\alpha : \alpha < \tau\}$ of size τ associated to $\mathcal{I}$ and the following two conditions are fulfilled:

(3) Any maximal chain contained in the union $\bigcup \Gamma$ meets each family v_α and has empty intersection.
(4-γ) If $\alpha < \beta$ and $V \in v_\alpha$, then the difference $V \setminus \bigcup v_\beta$ has cardinality γ.

Lemma 4.2. *(Assume CH) Let a poset $(\mathcal{R}, \subseteq)$ have tree property and be a MB-representation for an ideal $\mathcal{I}$. If for any two incompatible members $U, V \in \mathcal{R}$ it holds that $U \cap V \in \mathcal{I}$, then the ideal $\mathcal{I}$ has the ideal type $(\omega_1, \omega_1, \omega_1)$.*

Proof. Note first that $\mathcal{I}$ is a σ-ideal, which follows from the fact that $\mathcal{R}$ is σ-closed. Let $u = \{U_\alpha : \alpha < \omega_1\}$ be an antichain in $(\mathcal{R}, \subseteq)$. Then the set $N_\alpha = \bigcup\{U_\alpha \cap U_\beta : \beta < \alpha\}$ is in $\mathcal{I}$ for any $\alpha < \omega_1$. Thus from each $U \in u$ we can remove a set N_U from the ideal $\mathcal{I}$ in such a way that sets $U \setminus N_U$ and $V \setminus N_V$ are disjoint for distinct $U, V \in u$. Let $\Theta = \{u_\alpha : \alpha < \omega_1\}$ be a base tree for $(\mathcal{R}, \subseteq)$, and $U \in u_\alpha$. Then for each $\beta < \alpha$ there exists a unique member of u_β that contains U, so $E_U = \bigcup\{N_V : V \in \bigcup \Theta \text{ and } V \supseteq U\}$ is a member of $\mathcal{I}$, too. Thus the family of all sets of the form $U \setminus E_U$, where $U \in \bigcup \Theta$ has the property that any two of its members are either disjoint or one is contained in the other. Consider $\Gamma = \{v_\alpha : \alpha < \omega_1\}$, where $v_\alpha = \{U \setminus E_U : U \in u_\alpha\}$; it is a family that witnesses that $\mathcal{I}$ has the ideal type $(\omega_1, \omega_1, \omega_1)$.

We shall check conditions (1), (2), (3) and (4-ω_1) of the definition of the ideal type from above. Indeed, let $\mathcal{N}$ be the family of all nowhere dense sets on $\bigcup \mathcal{I}$ in the topology generated by the family $\bigcup \Gamma$. Note that $\bigcup \Theta$ is a MB-representation for $\mathcal{I}$ and $\bigcup \Gamma$ is a MB-representation for $\mathcal{N}$ and, moreover, $\bigcup \Gamma$ is a refinement of $\bigcup \Theta$ by which we mean that $\bigcup \Gamma$ refines $\bigcup \Theta$ and each member of the latter family contains a member of the former family. Moreover, each member of the family $\bigcup \Gamma$ contains a member from $\mathcal{R}$. These facts yield easily that $\mathcal{N} = \mathcal{I}$. Now observe that if A is a maximal antichain in $\bigcup \Gamma$ then the set $\bigcup \mathcal{I} \setminus \bigcup A$ is a member of $\mathcal{I}$, which follows from the fact that $\bigcup \Gamma$ is a MB-representation for $\mathcal{I}$. Additionally, if $X \in \mathcal{I}$, then the family $\{U \in \bigcup \Gamma : U \cap X = \emptyset\}$ is dense in $(\bigcup \Gamma, \subseteq)$, so any maximal antichain in this family is a maximal antichain, therefore $\bigcup \mathcal{I} \setminus \bigcup\{U \in \bigcup \Gamma : U \cap X = \emptyset\}$ is a member of $\mathcal{I}$, too. Hence, conditions (1) and (2) of the definition of the ideal type are fulfilled.

Let $\{X_\alpha : \alpha < \omega_1\}$ be a family of sets from $\mathcal{I}$ which covers $\bigcup \mathcal{I}$. We may assume that in the outset we have chosen a base tree Θ in such a way that always $\bigcup u_\alpha \cap X_\alpha = \emptyset$, which gives that any maximal chain in $(\bigcup \Gamma, \subseteq)$ has to have empty intersection, so the condition (3) is fulfilled. To see that also (4-ω_1) holds, observe first that we may take an injection $s : \bigcup \Theta \to \bigcup \mathcal{I}$ such that always $s(V) \in V \setminus E_V$ for $V \in \bigcup \Theta$. Moreover, we could have augmented sets E_U by an at most countable set $\{s(V) : V \in \bigcup \Theta \text{ and } V \supseteq U\}$ so as to ensure that the difference $(X \setminus E_X) \setminus (Y \setminus E_Y)$ contains an element from Y for any $X, Y \in \bigcup \Theta$ and $Y \subset X$. This amendment is sufficient to fulfill the last condition because each member of the tree $(\bigcup \Theta, \supseteq)$ contains ω_1 immediate successors, which completes the proof. □

5 Conclusions and questions

In [6] it was shown that under the hypothesis $t = h$ the ideal of nowhere Ramsey sets (r^0) has the ideal type $(2^\omega, h, 2^\omega)$, moreover in [5] it was shown that under the hypothesis $h = \omega_1$ the ideal (v^0) has the ideal type $(2^\omega, h, 2^\omega)$, too. Assuming CH and using Propositions 3.1, 3.2 and Fact 4.1 we get that premises of the previous Lemma are fulfilled, so we obtain that under CH the ideal (d^0) has the ideal type $(\omega_1, \omega_1, \omega_1)$. So, by the fact that the ideal type characterizes an ideal up to isomorphism, we obtain our main result:

Theorem 5.1. *(Assume CH) The ideals (d^0), (v^0) and (r^0) are isomorphic.*

The space of the rational numbers in the definition of (d^0) can be substituted for any countable space with a countable base in order to determine its ideal type to be $(\omega_1, \omega_1, \omega_1)$ under CH. The only one point on which we needed the rational numbers was homogeneity of $\mathbb{D}^*$, where we applied Sierpinski characterization of $\mathbb{Q}$. Nevertheless, under CH homogeneity of $(\mathbb{D}^*, \subseteq)$ is obvious and we do not need this characterization. So we can pose questions:

Q1 For which countable spaces X does the poset $(\mathbb{D}^*(X), \subseteq)$ (where X is taken instead of $\mathbb{Q}$ in the definition of $\mathbb{D}$) have tree property?

Q2 Are the ideals (v^0) and (d^0) isomorphic under some weaker assumption than CH (possibly in ZFC)?

References

[1] Balcar B., Hernández-Hernández F., Hrušák M., *Combinatorics of dense subsets of the rationals*, Fund. Math., 2004, 183, 59–80

[2] Balcar B., Pelant J., Simon P., *The space of ultrafilters on N covered by nowhere dense sets*, Fund. Math., 1980, 110, 11–24

[3] Brendle J., *Strolling through paradise*, Fund. Math., 1995, 148, 1–25

[4] Halbeisen L., *Making doughnuts of Cohen reals*, Math. Log. Quart., 2003, 49, 173–178

[5] Kalemba P., Plewik Sz., Wojciechowska A., *On the ideal* (v^0), Cent. Eur. J. Math., 2008, 6, 218–227

[6] Plewik Sz., *Ideals of nowhere Ramsey sets are isomorphic*, J. Symbolic Logic,1994 , 59, 662–667

[7] Repický M., *Collapsing of cardinals in generalized Cohen's forcing*, Acta Universitatis Carolinae, Mathematica et Physica 1988, 29, 67–74